Urban Consolidation Center. Formulations

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Abstract

Document that summarizes the formulations for UCC based on articles:

- [4] Mireia Roca-Riu and Miquel Estrada. An evaluation of urban consolidation centers through logistics systems analysis in circumstances where companies have equal market shares. Procedia - Social and Behavioral Sciences, 39:796 806, 2012. Seventh International Conference on City Logistics which was held on June 7- 9, 2011, Mallorca, Spain.
- [5] Mireia Roca-Riu, Miquel Estrada, and Elena Fernández. An evaluation of urban consolidation centers through continuous analysis with non-equal market share companies. Procedia - Social and Behavioral Sciences, 2016. Ninth International Conference on City Logistics which was held on June 17- 19, 2015, Tenerife, Spain.

Keywords: urban consolidation center, ...

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1. Introduction

The purpose of this work is to propose a compact model to quantify the effects of the implementation of a Urban Consolidation Center (UCC) to regulate urban distribution in an area of a city and to study under which circumstances it is globally beneficial for all participants. To do so, we propose a methodology which determines accurate approximations of all the costs involved.

The main idea behind our approach is that during certain time periods (preferably at night), carriers from all companies will bring the goods to the center with the possibility of using larger vehicles. During the day, with higher costumer density due to the consolidation of demand from several carriers, local deliveries will be performed more efficiently by a neutral freight carrier.

2. Assumptions and problem description

To easily describe the service area, we assume that several parameters of the zone are homogeneous, such as zone dimension, demand density, truck capacities, distance to closest depot, unit costs, number of carriers, and location of the center, among others. We further assume that the

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center will not be a warehouse, so goods will be received and shipped every day. We finally assume that carriers are willing to collaborate and use an UCC to perform last-mile deliveries.

The key definition variables are the dimensions of the vehicle zone delivery partition, which is assumed to be rectangular, vehicle load and the number of the trips per vehicle in the time horizon. The resulting cost decision function, traveled distance and vehicles-hour, can be simply formulated from the above variables and the parameters. Then, the model will be able to predict costs related to time and distance and any other interesting system metric.

We use the methodology of Logistic System Analysis [1] to define an accurate model for predicting costs and benefits of UCCs. Continuous approximation models produce robust solutions, particularly useful when dealing with strategic problems. This approach and the use of sensitive analysis allows us to study general tendencies and give more insight about the solutions.

In the following section, a basic model is formulated to find the optimal strategy that one company alone uses to serve from a depot, its clients spread over a delimited area. Then, the model is used as a basis to formulate and compare two alternative scenarios, (A) the independent transport companies performing last-mile delivery without UCC, and (B) a last-mile delivery system with collaboration among companies and consolidation through a UCC. In Section 2.2, we compare both situations where we further assume that companies have the same market share in the region, i.e. the number of customers per company is the same. Then, we relax the assumption in Section 2.3, and study the effect of non-homogeneous market-share. In Section 2.3.2, we model the trade-off between the participation of small carriers and its contribution to the savings of the UCC. The model can be used to determine a threshold on the minimum demand of a company to produce minimum desired savings in the operation costs of the consolidation center system.

2.1. General Model Description

Let us assume that one company has to serve N customers located in a rectangular zone of area A from a depot located at distance ρ from the center of the service area, with vehicles of capacity C. Let δ be the customers density. The company designs the tours with the objective of minimizing a weighted sum of distance costs (minimum tours) and temporal costs (minimum time consumption). As typical routing strategies do, first the region is partitioned in groups of approximately S points each. Note that S will be limited by time and capacity constraints. Then vehicle tours within the time horizon are designed. Vehicles travel from the depot to some point in its zone, serve the clients and return to the depot. We will call *line-haul* distance to the distance from the depot to the nearest point in its zone plus the distance from the last visiting point to the depot. And *local distance* is the distance covered during the delivery of the items. Let m be the average number of vehicles trips. Given the density of streets in urban areas we use the square grid metric to determine distances in the service area. Thus, S points should be located in a connected area. We assume that zones are rectangles with sides $2w$ and P, which form a partition of the whole delivery region. We next compute the total distance traveled to give service to a costumer by computing two different components: local and line-haul. The local traveled distance is independent of the point where the route starts or ends the delivery. [3] proposed a simple (non-optimal) strategy for visiting the points in each zone and showed that if we use nearly rectangular partitions of the region, the partitions should be elongated towards the depot. If vehicles carry a full load, the number of points in the rectangle should be C, which expressed in terms of density should be equivalent to $2wP\delta$. However, we might be interested in reducing the average number of points to S. So we will denote the number of expected customers inside a rectangle as S and use the equality $2wP\delta = S$ to eliminate one variable. To estimate the local length of a tour as a function of w and S, we extend Daganzo's proposal [1] and divide the rectangle into two bands each of width w. Then, each route visits points in non-decreasing coordinate x along the length of the rectangle on the way out and decreasing x on the way back (See Figure 1). If points are randomly distributed in space, one can evaluate the expected total distance. We divide the distance into the traverse and the longitudinal, since we use square grid metric to approximate the distance. The average traverse travel distance per point is simply the average distance between two random points on an interval of width w, that is $w/3$. The total longitudinal travel in the rectangle is $2P$ or $2P/S$ per point and using the equality $2wP\delta = S$, we obtain $1/\delta w$. The average line-haul distance for a vehicle will be $2\rho - P$ or $((2\rho - P))/S$ per point. We subtract P to the distance of the depot due to the relative position of the service areas with the depot and, again replacing P using the equality, we obtain $((2\rho - S/2\delta w))/S$. Finally, adding the line-haul and local distances we obtain an approximation of the total distance traveled per point:

$$d = \frac{w}{3} + \frac{1}{\delta w} + \frac{2\rho - \frac{S}{2\delta w}}{S}.$$

Apart from costs derived from the covering of distance, costs also come from the time spent during the delivery. Using v_A,v_B and τ as urban speed inside the service area, interurban speed from the depot to the service area, and time lost per customer's unloading/loading of a vehicle respectively, the total time per vehicle trip is:

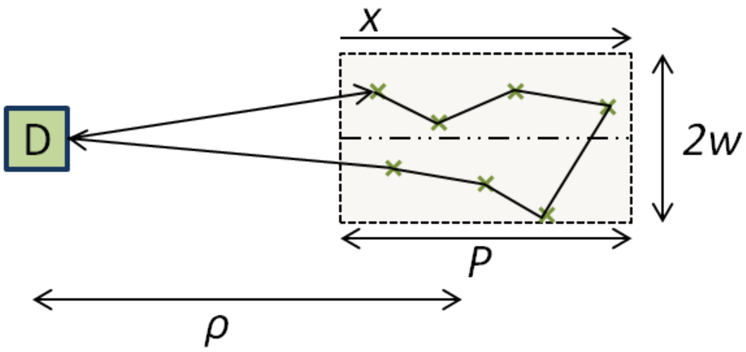

Figure 1: Local Strategy to cover all points

$$\frac{2\rho - P}{v_B} + \frac{2P}{v_A} + \frac{wS}{3v_A} + \tau S.$$

Then, the time devoted to each customer can be obtained from the above equation, dividing by the customers served by one vehicle (S).

$$\frac{2\rho - P}{v_B S} - \frac{1}{2w\delta v_B} + \frac{1}{w\delta v_A} + \frac{w}{3v_A} + \tau.$$

The objective function (1) is obtained as a weighted sum of distance costs and time related costs, with unit cost parameters c_d and c_t, . The resulting model is, thus:

$$\min_{w,S,m} \quad N\left[\left(c_d+\frac{c_t}{v_B}\right)\frac{2\rho}{S}+\left(\frac{c_d}{2}+\frac{c_t}{v_A}-\frac{c_t}{2v_B}\right)\frac{1}{w\delta}+\left(c_d+\frac{c_t}{v_A}\right)\frac{w}{3}+c_t\tau\right] \tag{1}$$

$$S \leq C \tag{2}$$

$$\frac{2\rho}{v_B}-\frac{S}{2w\delta v_B}+\frac{S}{w\delta v_A}+\frac{wS}{3v_A}+\tau S \leq \frac{Y}{m} \tag{3}$$

$$w, S, m \geq 0 \qquad \text{continuous} \tag{4}$$

Constraints (2)–(3) model the limitations on the capacity of the vehicles, C, or on the time horizon, Y. We can solve the problem (1)–(4) analytically due to the nature of its variables. As m can become as small as needed, constraint (3) can be omitted making m equal to $Y/({}^{2\rho}/_{v_B} - {}^{S}/_{2w\delta v_B} + {}^{S}/_{w\delta v_A} + {}^{wS}/_{3v_A} + \tau S)$ which is, obviously, positive. Then, in order to minimize the objective function, S should be as big as possible, which makes S equal to C, and from minimum conditions we obtain $w^* = \sqrt{(3({}^{c_d}/_2 + {}^{c_t}/_{v_A} - {}^{c_t}/_{2v_B})/\delta(c_d + {}^{c_t}/_{v_A}))}$. Let us call $\lambda = (({}^{c_d}/_2 + {}^{c_t}/_{v_A} - {}^{c_t}/_{2v_B}))/((c_d + {}^{c_t}/_{v_A}))$, and so $w^* = \sqrt{{}^{3\lambda}/_{\delta}}$. The most important metrics are summarized in Table 1.

Local distance	$D_L = N\frac{\lambda+3}{3(\delta\lambda)^{1/2}}$
Line-haul distance	$D_{LH} = N\left[\frac{2\rho}{C} - \frac{(3\lambda)^{1/2}}{2\delta^{3/2}}\right]$
Time	$T = N\left[\frac{2\rho}{v_B C} + \frac{1}{(3\lambda\delta)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta}\right)^{1/2}\frac{1}{v_A} + \tau\right]$
Cost	$c_d\left(D_L + D_{LH}\right) + c_t T$

Table 1: Summary of the most important metrics of the general model

The costs of the system are proportional to N, the total number of customer of the system. The square root of customer density is dividing the terms in the total cost expression regarding local distribution, whereas capacity is dividing the terms regarding line-haul distribution. Finally, there is a constant time cost of stopping.

2.2. *UCCs with equal market share companies*

In this section two models are proposed for each type of delivery strategy. The models approximate the costs of both situations: (A) the total costs of the system where each company performs their last-mile delivery independently, and, (B) the costs of a system where the companies collaborate and consolidate demand through a UCC. We assume that each company has the same market share in the delivery area. In the next section, this assumption is relaxed.

We assume that $M \geq 1$ equal-market share companies give service to N costumers, with the same number of customers each $\widehat{N} = N/M$. Therefore, $\widehat{\delta} = \delta/M$, which is smaller than δ. In the collaborating strategy, we consider that companies can use bigger trucks with capacity $B = k_C C$ ($k_C \geq 1$), and that the UCC is located at distance φ from the center of the service region ($\varphi = k_\rho \rho$ with $k_\rho \leq 1$). Note that k_C represents the enlargement of vehicle capacity in the line-haul distribution. We will call *capacity enlargement* to this parameter. Similarly, k_ρ represents the reduction of the distance from the closest depot to the final destinations. We call *depot distance reduction* to this parameter.

The two strategies will be compared in terms of costs, distance, and time consumption, assuming that all the companies try to minimize their costs. As mentioned before, in the system without a UCC it is assumed that each company carries the distribution to its customers independently with its own fleet. The total costs are the sum of the costs of each individual company. On the contrary, in the UCC system it is assumed that the costs are split in two delivery phases: the costs that each company undergoes to bring the goods to the consolidation center with its own fleet and the costs of the neutral freight carrier for last-mile distribution. It is further assumed that the distribution center is not a warehouse and that goods are received and shipped on the same day. Thus, no holding costs are incurred. Table 2 summarizes the results for strategy A, when each transport operator acts independently from the others, whereas Table 3 shows the results for strategy B, when operators act in collaboration through the use of a UCC.

Local distance	$D_{LA} = M\widehat{N}\frac{\lambda+3}{3\left(\widehat{\delta}\lambda\right)^{1/2}} = N\frac{\lambda+3}{3\left(\frac{\delta}{M}\lambda\right)^{1/2}} = M^{1/2}N\frac{\lambda+3}{3(\delta\lambda)^{1/2}}$
Line-haul distance	$D_{LHA} = M\widehat{N}\left[\frac{2\rho}{C} - \frac{(3\lambda)^{1/2}}{2\widehat{\delta}^{3/2}}\right] = N\frac{2\rho}{C} - M^{3/2}N\frac{(3\lambda)^{1/2}}{2\delta^{3/2}}$
Time	$T_A = M\widehat{N}\left[\frac{2\rho}{v_B C} + \frac{1}{\left(3\lambda\widehat{\delta}\right)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\widehat{\delta}}\right)^{1/2}\frac{1}{v_A} + \tau\right] =$ $N\left[\frac{2\rho}{v_B C} + M^{1/2}\left(\frac{1}{(3\lambda\delta)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta}\right)^{1/2}\frac{1}{v_A}\right) + \tau\right]$
Cost	$c_d(D_{LA} + D_{LHA}) + c_t T_A$

Table 2: Summary of for the model without consolidation (strategy A)

The critical parameters to decide if the consolidation is benefitting in terms of distribution costs are: the number of companies collaborating, the capacity enlargement and the depot distance. It is clear that the total local distance is reduced by the proportion of the square root of the companies participating. The line-haul distance has two main components. The first one has been computed for different values of k_C and k_ρ. If $(k_\rho + 1/k_C) = 1$ the first line-haul distance component is kept. If the value is less than one, more savings can be obtained proportionally to this value. The second component is negligible as it is one order of magnitude less than the first one. Distribution time has a more complex formulation than distance; in fact, time is a reformulation of distance with speed parameters. In any case, most of the distance is traveled with interurban speed, so time is also reduced in the model for consolidation.

Local distance	$D_{LB} = N\frac{\lambda+3}{3(\delta\lambda)^{1/2}}$
Line-haul distance	$D_{LHB} = M\widehat{N}\frac{2\rho}{B} + N\left[\frac{2\varphi}{C} - \frac{(3\lambda)^{1/2}}{2\delta^{3/2}}\right] =$ $= N\frac{2\rho}{k_C C} + N\frac{2k_\rho \rho}{C} - N\frac{(3\lambda)^{1/2}}{2\delta^{3/2}} =$ $= N\frac{2\rho}{C}\left(k_\rho + \frac{1}{k_C}\right) - N\frac{(3\lambda)^{1/2}}{2\delta^{3/2}}$
Time	$T_B = N\left[\frac{2\rho}{v_B B} + \frac{2\varphi}{v_B C} + \frac{1}{(3\lambda\delta)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta}\right)^{1/2}\frac{1}{v_A} + \tau\right] =$ $= N\left[\frac{2\rho}{v_B C}\left(k_\rho + \frac{1}{k_C}\right) + \frac{1}{(3\lambda\delta)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta}\right)^{1/2}\frac{1}{v_A} + \tau\right]$
Cost	$c_d(D_{LB} + D_{LHB}) + c_t T_B$

Table 3: Summary of metrics for the model with consolidation (strategy B)

2.3. UCCs with non-equal market share companies

Similarly to the previous section, we assume there is a delivery zone of area A, where M companies operate and a total of N customers are uniformly distributed within the area A. We now assume that each company gives service to N_i customers, $i = 1, \ldots, M$, not necessarily equal, with $N = \sum_{i=1,..,M} N_i$. The customer density of each company is denoted by $\delta_i = N_i/A$, $i = 1, \ldots, M$. Note that δ_i is smaller than the overall demand density $\delta = N/A$. We use the same parameters as in the previous section. The distance from the depot to the geographical center of the service area is denoted by ρ. We further assume a fixed capacity C for the vehicles that perform urban distribution. In the case where no UCC exists, those vehicles traveling from the depot to the service area also have capacity C. Instead, we assume that companies use (bigger) trucks with capacity $B = k_C C$, ($k_C \geq 1$) when they use the consolidation center. The parameter k_C represents the relative increment of vehicle capacity in line-haul distribution and it is called *capacity enlargement.* We further assume that the UCC is located at distance φ from the geographic center of the service region with $\varphi = k_\rho \rho$, $k_\rho \leq 1$, i.e. the UCC is closer than the company's depot to the geographic center of the service region. The parameter k_ρ is called *depot distance reduction.* It relates two distances: the distance ρ from the geographical center of the service area to the depot of the carrier and the distance φ from the geographical center of the service area to the UCC.

Next, we present with the same methodology as in the previous section a continuous model for different market share distributions, which allows again the comparison of two alternative urban delivery strategies, when carriers have non-equal market-shares: A) a system without the UCC and B) a system with the UCC. Both strategies are compared in terms of costs, distance, and time consumption, under the assumption that all the companies are trying to minimize their costs. In strategy A the total costs are the sum of the costs of each individual company. On the contrary, in strategy B it is assumed that the costs are split in two delivery phases: the costs that each company undergoes to bring the goods to the consolidation center with its own fleet and the costs of the neutral freight carrier for last-mile distribution. Again, no holding costs are incurred since the distribution center is not a warehouse. Table 4 summarizes the results for strategy A, when each transport operator acts independently from the others, whereas Table 5 shows the results for strategy B, when operators act in collaboration through the use of a UCC. Note that we use D_{LHA}, D_{LA} and T_A, and D_{LHB}, D_{LB} and T_B to refer to the metrics of Strategy A and B respectively, but in this case with non-equal market share company.

When analyzing the potential benefit of a consolidation center under the non-equal market share assumption, the new critical aspects in terms of distribution costs are the number of collaborating

Local distance	$D_{LA} = \frac{\lambda+3}{3\lambda^{1/2}} \sum_{i=1}^{M} \frac{N_i}{\delta_i^{1/2}}$
Line-haul distance	$D_{LHA} = N\frac{2\rho}{C} - \frac{(3\lambda)^{1/2}}{2} \sum_{i=1}^{M} \frac{N_i}{\delta_i^{3/2}}$
Time	$T_A = N\left[\frac{2\rho}{v_B C} + \tau\right] + \sum_{i=1}^{M} \frac{N_i}{\delta_i^{1/2}} \left(\frac{1}{(3\lambda)^{1/2}} \left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta}\right)^{1/2} \frac{1}{v_A}\right)$

Table 4: Summary of for the model without consolidation (strategy A)

Local distance	$D_{LB} = \frac{\lambda+3}{3\lambda^{1/2}} \frac{N}{\delta^{1/2}}$
Line-haul distance	$D_{LHB} = N\frac{2\rho}{C}\left(k_\rho + \frac{1}{k_C}\right) - \frac{(3\lambda)^{1/2}}{2} \frac{N}{\delta^{3/2}}$
Time	$T_B = N\left[\frac{2\rho}{v_B C}\left(k_\rho + \frac{1}{k_C}\right) + \tau\right] + \frac{N}{\delta^{1/2}} \left[\frac{1}{(3\lambda)^{1/2}} \left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3}\right)^{1/2} \frac{1}{v_A}\right]$

Table 5: Summary of metrics for the model with consolidation (strategy B)

companies and the market share distribution, i.e. which portion of the customers has each of the carriers.

2.3.1. Analysis for several market share distributions

Some data of transport companies registered in Barcelona (Spain) was analyzed to check the heterogeneity of real markets [2]. Transport companies were classified by sales revenue and the number of employees, which are two variables closely related to market share. Figure 2 depicts the number of companies in each class . We can observe that the more significant group was the one of small companies, i.e., the group with smallest values both in sales and employees.

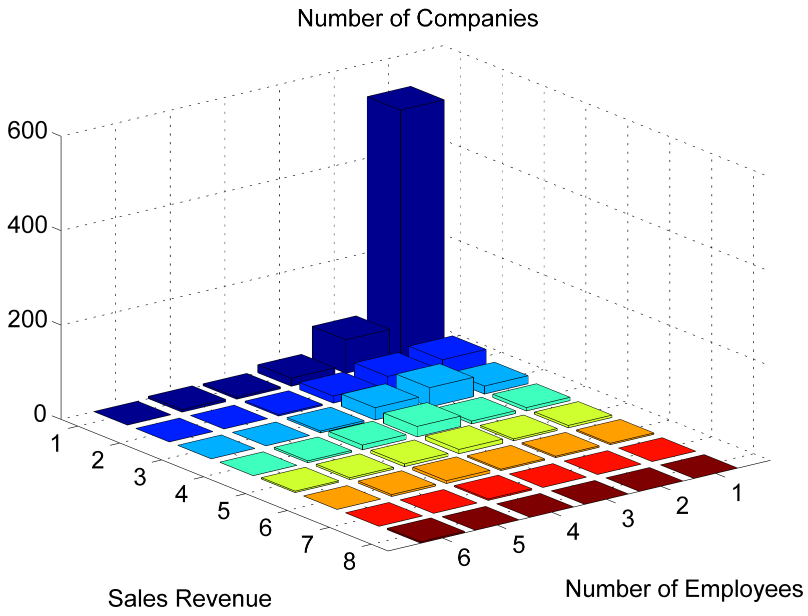

Figure 2: Number of transport companies in Barcelona (Spain) classified by sales revenue and number of employees

The difficulty to obtain reliable data of market share in a particular area lead us to analyze different possible distributions. In the PhD thesis [?], we formulate some possible market distributions based on some analytically simple functions. Since we cannot guarantee that in reality

Sales revenue (in millions €)	**Number of employees**
1-until 0.3 €	1-from 1 to 5
2-from 0.3 to 0.6 €	2-from 6 to 10
3-from 0.6 to 1.5 €	3-from 11 to 25
4-from 1.5 to 3 €	4-from 26 to 50
5-from 3 to 6 €	5-from 51 to 100
6-from 6 to 15 €	6-from 101 to 250
7-from 15 to 30 €	
8-from 30 to 60 €	

Table 6: Legend of Figure 2

the market adjusts to some known functions, also in the thesis an extended set of distributions is proposed, which covers any possible distribution of customers among carriers.

Experimentally, we compared the four simple market share distributions. The total costs are similar in the four market share distributions, then savings with UCC are also similar. We observe that the univrom distribution is the one that provides more savings, but the maximum difference with the others is smaller than 0.5%.

Also experimentally, the extended set of potential market distributions for N customers among M companies is also analyzed. We generated all marked distributions with an algorithm and analyzed the different potential savings in percentage that they provide compare to the non-sharing situation. The savings range between 6.3% and 7% among the different distributions. The differences in savings are not significant, so we can conclude that market structure does not significantly affect savings in UCC.

2.3.2. Trade-off between minimum carrier dimension and savings

Due to a highly competitive market, a system with a UCC could be objected by medium-size and large carriers. The reason for which large carriers would like to limit the participation of very small carriers is that small carriers do not significantly increase the number of customers, but are greatly benefitted from the consolidation center. In the thesis [**?**] we present a tool to determine a threshold on the minimum demand of a company to join the UCC.

Experimentally, we can build a range of savings based on a parameter that relates the number of costumers of one carrier that joins the UCC and the size of the UCC customers. In particular, with this, we can decide which is the minimum percentage reduction of the unit cost per costumer to accept an incorporation of a new carrier. Potential savings range from 5% if the independent carrier has 5% of the customers of the consolidation center, and up to a 32.5% if the independent carrier brings 50% of the demand of the UCC.

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